Communication for maths

On the formal presentation of the binomial theorem

Focusing on the binomial theorem.

Correct use of symbols

- Use the equal sign "=" and the ellipsis sign "…" or the approximately equals sign "≈" appropriately

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2$$

No

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$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2$$

No

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2 + \cdots$$

Yes

$$(1+x)^{\frac{1}{2}} \simeq 1 + \frac{1}{2}x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2$$

Yes

Focusing on the binomial theorem.

Use the method stated

Using the binomial theorem expand $(1 + x)^4$

$$(1+x)^4 = \binom{4}{0} + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + \binom{4}{4}x^4$$

or

$$(1+x)^4 = 1 + 4x + \frac{4(3)}{2!}x^2 + \frac{4(3)(2)}{3!}x^3 + \frac{4(3)(2)(1)}{4!}x^4$$

Focusing on the binomial theorem.

Use the method stated

Using Pascal's triangle expand $(1 + x)^4$

Power of Binomial term			Coefficients							
0 :					1					
1 :				1		1				
2 :			1		2		1			
3 :		1		3		3		1		
4 :	1		4		6		4		1	

$$\therefore \quad (1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$

Focusing on the binomial theorem.

No free-standing expressions

Expand $(1+x)^{\frac{1}{2}}$ up to the

term in x^2.

Solution :

$$1 + \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2$$

$$1 + \frac{x}{2} - \frac{1}{8}x^2$$

Focusing on the binomial theorem.

Justification

Expanding $(1-x)^{1/2}$ upto, and including, the term x^3,

find $\sqrt{2}$ to 2 d.p.

Solution: $(1-x)^{1/2} \simeq 1 - \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3$

LHS: when $x = 0.02$, $(1-0.02)^{1/2} = \frac{7}{10}\sqrt{2}$

How so? Show steps

Focusing on the binomial theorem.

Justification

Expanding $(1-x)^{1/2}$ upto, and including, the term x^3,

find $\sqrt{2}$ to 2 d.p.

Solution: $(1-x)^{1/2} \simeq 1 - \frac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3$

LHS: when $x = 0.02$

the term $(1-0.02)^{1/2} = (0.98)^{1/2}$

$= \sqrt{\frac{98}{100}} = \sqrt{\frac{49 \times 2}{100}} = \frac{7}{10}\sqrt{2}$

Yes

Focusing on the binomial theorem.

Justification

RHS: when $x = 0.02$ we have

$$1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{3}{48}x^3 = 0.9899495$$

How so?
Show steps

Focusing on the binomial theorem.

Justification

RHS : when $x = 0.02$ we have

$$1 - \frac{1}{2}(0.02) - \frac{1}{8}(0.02)^2 - \frac{3}{48}(0.02)^3 \simeq 1 - 0.01 - 0.00005 - 0.0000005$$

$$= 0.9899495$$

Yes

Focusing on the binomial theorem.

Justification

Or

Yes

Let $g(x) = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{3}{48}x^3$

$\therefore \; g(0.02) = 0.9899495$

Focusing on the binomial theorem.

Justification

Justification steps demonstrate your mathematical understanding of why a future step is what it is.

Focusing on the binomial theorem.

- **Exact versus approximate values**
  - Exact value: $\sqrt{2}$, 2π, …
  - Approximate values: 1.41 to 2 d.p.; 6.283 to 3 d.p.
  - Decimal place accuracy:
    - If a final answer is required to 6 d.p. then work to at least 7 d.p. throughout the whole of the solution.
    - Only present your final answer to 6 d.p. not any intermediate results.

Focusing on the binomial theorem.

Example:

Expanding $(1 + 3x)^{1/2}$ by the binomial theorem, up to and including the term in x^3, find √7 to 5 d.p.

Answer

Note that the correct answer is $\sqrt{7} \approx 2.64577$

Solution

See next slide. There are two presentation errors in the solution below, one relating to decimal places. Can you find the other error?

Solution : Given $f(x) = (1+3x)^{1/2}$

we have $f(0.04) = (1 + 3(0.04))^{1/2}$

$$= \frac{\sqrt{28}}{5} = \frac{2}{5}\sqrt{7}.$$

Also $f(x) \simeq 1 + \frac{3}{2}x + \frac{3/2\,(3/2 - 1)}{2!}x^2$

$$+ \frac{3/2\,(3/2-1)(3/2-2)}{3!}x^3$$

$$\simeq 1 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{27}{16}x^3$$

$\therefore$ $f(0.04) \simeq 1.0583$

So $\sqrt{7} \simeq \frac{5}{2}(1.0583) \simeq 2.64575$

Focusing on the binomial theorem.

Align your equals signs

The Binomial term

$$(1-0.02)^{\frac{1}{2}} = (0.98)^{\frac{1}{2}}$$

$$= \sqrt{\frac{98}{100}} \qquad = \sqrt{\frac{49 \times 2}{100}}$$

$$= \frac{7}{10}\sqrt{2}$$

Focusing on the binomial theorem.

Align your equals signs

The Binomial term

$$(1-0.02)^{1/2} = (0.98)^{1/2}$$

$$= \sqrt{\frac{98}{100}} \qquad = \sqrt{\frac{49 \times 2}{100}}$$

$$= \frac{7}{10}\sqrt{2}$$

Not aligned or correctly spaced

Focusing on the binomial theorem.

Do not write in columns

LHS: when $x = 0.02$ we have

$$(1-0.02)^{1/2} = (0.98)^{1/2}$$

$$= \left(\frac{98}{100}\right)^{1/2}$$

$$= \left(\frac{49\times 2}{100}\right)^{1/2}$$

$$= \frac{7}{10}\sqrt{2}$$

$$\ldots$$

For RHS we have

$$1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{3}{48}x^3$$

$$\simeq 1 - \frac{1}{2}(0.02) - \frac{1}{8}(0.02)^2 - \frac{3}{48}(0.02)^3$$

$$\simeq 1 - 0.01 - 0.000005 - 0.0000005$$

$$\ldots$$

Example 1

Consider the following question

Let

$$(a + bx)^n = C_n x^n + C_{n-1} x^{n-1} + \cdots + C_1 x + C_0.$$

Given $(3 + bx)^n$ find the possible values of b and n when

$C_0 = 243$ and $C_4 = 1280/2187.$

Let us now study the solution handed out to identify what makes this solution incomplete.

Exercise 1

Write a correct solution to the following problem

Find the coefficient of x^{115} in the expansion of

$$\left(4x^4 - \frac{5}{x^3}\right)^{55}.$$

Write your answer in terms of factorials, and powers of 2 and 5.

Presentation

Reminder

- The above slides refer to only a few of the aspects of mathematical presentation.
- Refer to previous slides for all other aspects of mathematical presentation in order to give a full and proper solution to a problem.

Presentation

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Appendix

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Polynomial terminology

- What is the basic polynomial vocabulary?

$$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0.$$

Leading term	Leading coefficient	Constant term	Coefficient of the x^3 term
Fourth term	Approximately equal to	Expanding up to the x^3 term	(x – 1) is a factor of
Factorising	The fourth term	To factorise / factorising	To expand / expanding

Polynomial phrasing examples

- See handout
- *Dividing: $6 \div 3$*
  - *"6 divides 3" or "3 divides 6"?*
  - *"6 is divided by 3" or "3 is divided by 6"?*
- *Equation vs inequality*
  - *$3x + 1 = 2$*
  - *$3x + 1 > 2$*
  - *$3x + 1$*

General vocabulary

- Linking terms or phrases still apply:

Hence	Therefore	So
Implies	Simplifying (we get)	Factorising (we obtain)
Dividing by … (we get)	Multiplying both sides by …	Comparing left and right hand sides
Substituting … we get …	Given that …	We see that …

General vocabulary

- Linking terms or phrases still apply:

For all …	There exists …	Such that …
The value …	Satisfies …	The exact value of …
The approximate value of … to 2 decimal places	Because (of) …	Since …
We have …	It follows that …	Let …